Assignment 8: Due Friday 31st May at 5pm

Late assignments will not be accepted except by prior arrangement (for a good reason)

Please include your student number in your handed up work, as Canvas doesn't give this to me automatically.

This task takes the form of a competition. Marks will be awarded based on how well your code performs. A small prize will be awarded to the team whose code performs best.

You may do this exercise individually or in teams of two. Please let me know what choice you make by **October 5th.**

Background

Take a sequence as follows. From an initial value $f(0)$ repeat the following:

$$f(n+1) = \begin{cases} \dfrac{x_n}{2}, & \text{if } n \text{ is even,} \\ 3n+1, & \text{if } n \text{ is odd.} \end{cases}$$

For instance, we might start at $f(0) = 12$ resulting in the sequence $12, 6, 3, 10, 5, 16, 8, 4, 2, 1$. I have stopped the sequence here because it now cycles through the sequence $4, 2, 1$ indefinitely.

Another example is the sequence $19, 58, 29, 88, 44, 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1$. Note that this sequence also ends in $4, 2, 1$.

This seemingly simple sequence is actually very interesting. The **Collatz Conjecture** states that for positive starting numbers this sequence will always (eventually) reach 1.

`https://en.wikipedia.org/wiki/Collatz_conjecture`

`https://arxiv.org/pdf/math/0309224.pdf`

The conjecture has been demonstrated to be true for very, very large numbers, but has not been proved despite many mathematicians trying.

It and its variations are known by many names: Ulam's conjecture, Kakutani's problem and hailstone numbers. Paul Erdős said "Mathematics may not be ready for such problems," and Jeffrey Lagarias stated in 2010 that the Collatz conjecture "is an extraordinarily difficult problem, completely out of reach of present day mathematics."

Task

In don't expect you to prove Collatz Conjecture, but I will award bonus marks if you do.

If true, the conjecture implies a tree that defines the pathways of all numbers to the root number 1.

For instance: the tree created by all number sequences of length 10 or fewer hops is shown in Figure 1.

The tree defines a *topological ordering* of its nodes with the additional criteria that when two nodes are the same distance from the root, we sort them in numerical order. Thus, for the tree in Figure 1 we obtain the ordering

$$1; 2; 4; 8; 16; 5, 32; 10, 64; 3, 20, 21, 128; 6, 40, 42, 256; 12, 13, 80, 84, 85, 512; 24, 26, 160, 168, 170, 1024;$$

In this sequence, numbers at the same topological distance are separated by commas and numbers at different distances by semi-colons.

The task is to create a topological ordering for the largest possible distance D from the root of the tree at 1. You should create a set of topological orderings with $D = 2, 3, ...$ up to the maximum value that your code can cope with.

You do not have to write your code in Julia. I will not be assessing your code.

Assessment

Your mark will be proportional to a function of D_{max}, which I will choose to provide a reasonable scale and discrimination.

I will autocheck your sequences and calculate D_{max} the highest value at which your code is correct.

Note that orderings up to D_{max} will contain orderings for smaller D, but including some smaller values will give you a backup result in case something breaks in your code for larger D.

What to Hand In

The main file you hand up should be a CSV file, in the form, of the distance, and then the **top 10** largest values in decreasing order. An example is included below:

```
D,1,2,3,4,5,6,7,8,9,10
7,128,21,20,3,64,10,32,5,16,8
10,1024,170,168,160,26,24,512,85,84,80
```

You should also hand up a brief PDF document describing the strategy you applied to the problem. You may want to include complexity analysis of your approach as well as well as details of the computational tools, e.g., hardware, that you used.

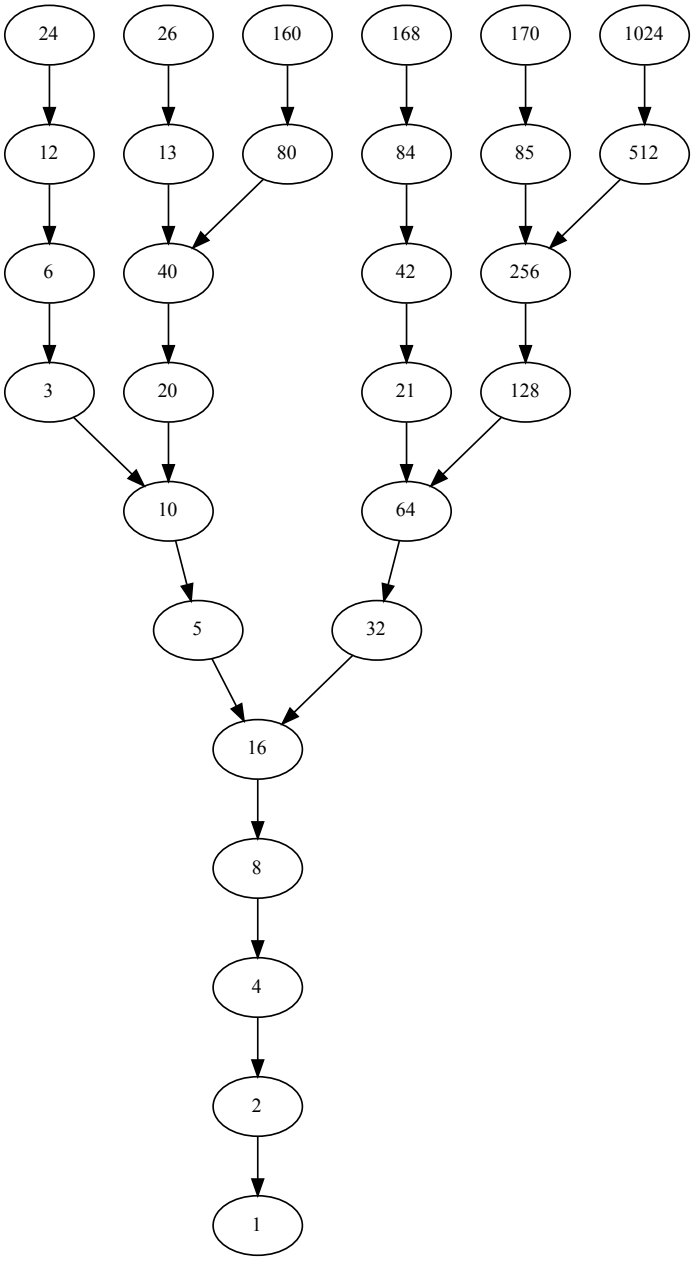

Figure 1: The tree defined by Collatz sequences of 10 hops or less.

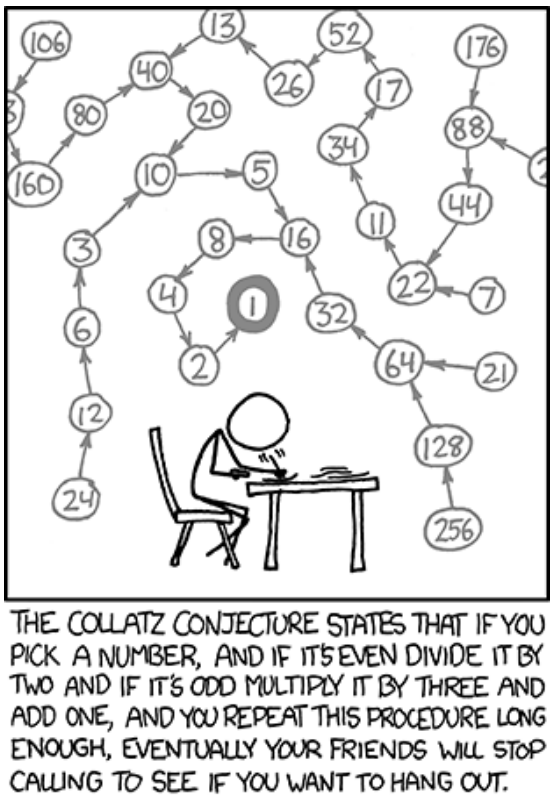

Figure 2: xkcd.com/710/