Tutorial 1

Make sure you prepare these BEFORE the class.

Solutions will be handed out at the tutorial. They will not be put on MyUni.

1. **Translation:** A farmer has three crops he can grow: wheat, rice and cotton. Each results in revenue (per acre) of \$100, \$300, \$200. However, the rice and cotton must be irrigated taking 110 kilolitres and 100 kilolitres of water per day, respectively, and the farmer has only 3000 kilolitres of water available per day for the whole farm. All must be fertilised, with respective costs of \$40, \$30 and \$20 per acre.

   Interrogate the problem and formulate an optimisation problem to tell the farmer how much of each crop to grow on his/her 50 acre farm to maximise his/her profits.

   Hints: remember to look for three things:

   1. the variables (the things you can control);
   2. the objective (the thing you want to maximise or minimise); and
   3. the constraints (there are 2 main constraints here, but don't forget non-negativity).

   Tabulate the data, and then construct the optimisation in standard form.

2. **Interpretation:** Imagine that we took a problem expressed in the form

$$\begin{aligned} \max \quad & z = \mathbf{c}^T \mathbf{x} \\ \text{subject to} \quad & A'\mathbf{x} \le \mathbf{b} \\ & \mathbf{x} \ge \mathbf{0} \end{aligned}$$

   and when we converted it into standard inequality form, we arrived at a Tableau for the equalities, on which we performed a series of row operations to obtain

$$\begin{array}{|ccccccc|} \hline 1 & 1 & 0 & 1 & 0 & 0 & 5 \\ 0 & 3 & 1 & 0 & -1 & 0 & 7 \\ 0 & 3 & 0 & 1 & -1 & 1 & 4 \\ \hline \end{array}$$

   (a) Write down a solution to this problem with three basic, and three non-basic variables (Hint: it should be possible to do so immediately).

   (b) Interpret this solution in the light of the optimisation problem specified in terms on inequalities above.

3. **Calculations:** Translate the following problem into standard equality form.

$$\begin{array}{lrcrcrcr} \min \ z = & 2x_1 & - & 2x_2 & + & 3x_3 & & \\ \text{subject to} & & & & & & & \\ & -x_1 & + & 2x_2 & + & x_3 & \le & 4 \\ & 2x_1 & - & x_2 & + & 2x_3 & \ge & -2 \end{array}$$

   with $x_1 \ge 0$, $x_2 \le 0$, and x_3 free.

   Hints: some tricks you will need:

   1. You need to convert it into a maximisation problem.
   2. You need to convert a $\ge$ constraint into a $\le$
   3. You need to swap a non-positive variable with a non-negative one.
   4. You need to replace a free variable with two non-negative variables.
   5. You need to add slack variables to convert the constraints into equalities.

4. **Proof of the week:**
   (a) Show that in $\mathbb{R}^3$, if three planes don't have any point where all three meet, then their governing equations have linearly dependent coefficients.
   (b) Describe the cases which allow this to happen.
   (c) Comment on the importance of these results for Linear Programming in the 3D case. Can you also generalise to higher dimensions?