

# **The Makerbot: Desktop 3D printing**

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# Star Trek Predictions

- Communicators => Mobile phones
- Personal Access Display Device  
=> iPad
- Transparent aluminium => aluminium oxynitride
- Replicators => ???

<http://www.youtube.com/watch?v=pzqW0YaN2ho>

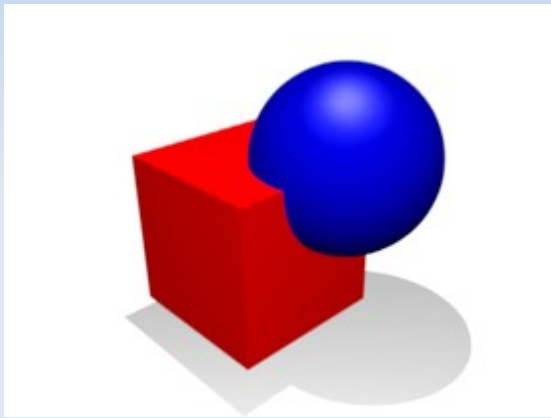
# CNC

- CNC = Computer Numerical Control
- CNC machines have been around for a while
  - 3D printers are one of many (e.g. Milling machines)
- Recently
  - Drop in price (as low as \$800 for a kit)
  - Improvement in how easy they are to build and run
- Various tech
  - Resin, lasers, etc.
  - Makerbot desposits layers of molten plastic

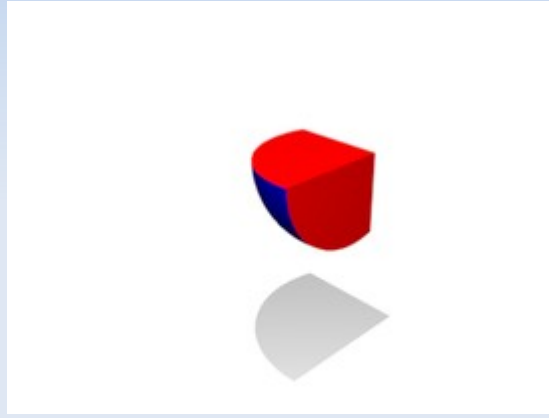
# Maths

- Lots of maths embedded
  - Control (temperature, position, ...)
  - Representation of objects
    - CSG = Constructive Solid Geometry
  - Algorithms for converting objects to tool-head path
    - Approximation of curves into straight lines
- Like much modern tech, the math is hidden

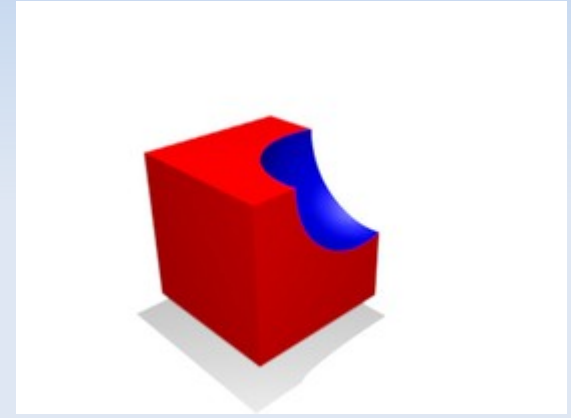
# CSG



Union



Intersection



Difference

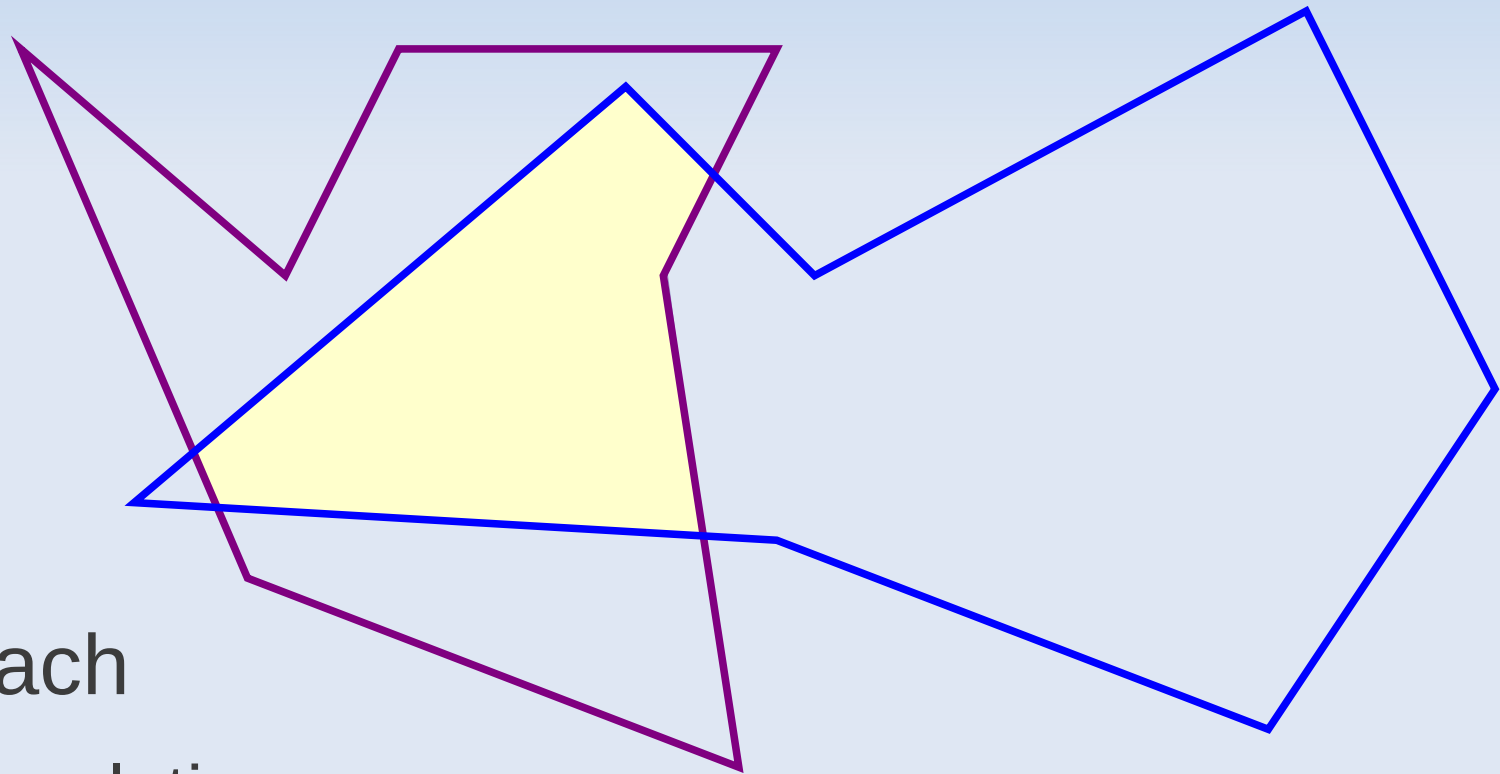
[http://en.wikipedia.org/wiki/Constructive\\_solid\\_geometry](http://en.wikipedia.org/wiki/Constructive_solid_geometry)

# Tool-head path

- Slide the object into horizontal layers
  - Compute intersections of plane with 3D object
- Determine how to fill in the physical region
  - Hollow interior
  - Strong boundary layer
- Approximate curves with straight line segments
- Most efficient to do all at once
  - e.g., compute intersections of triangles in 2D

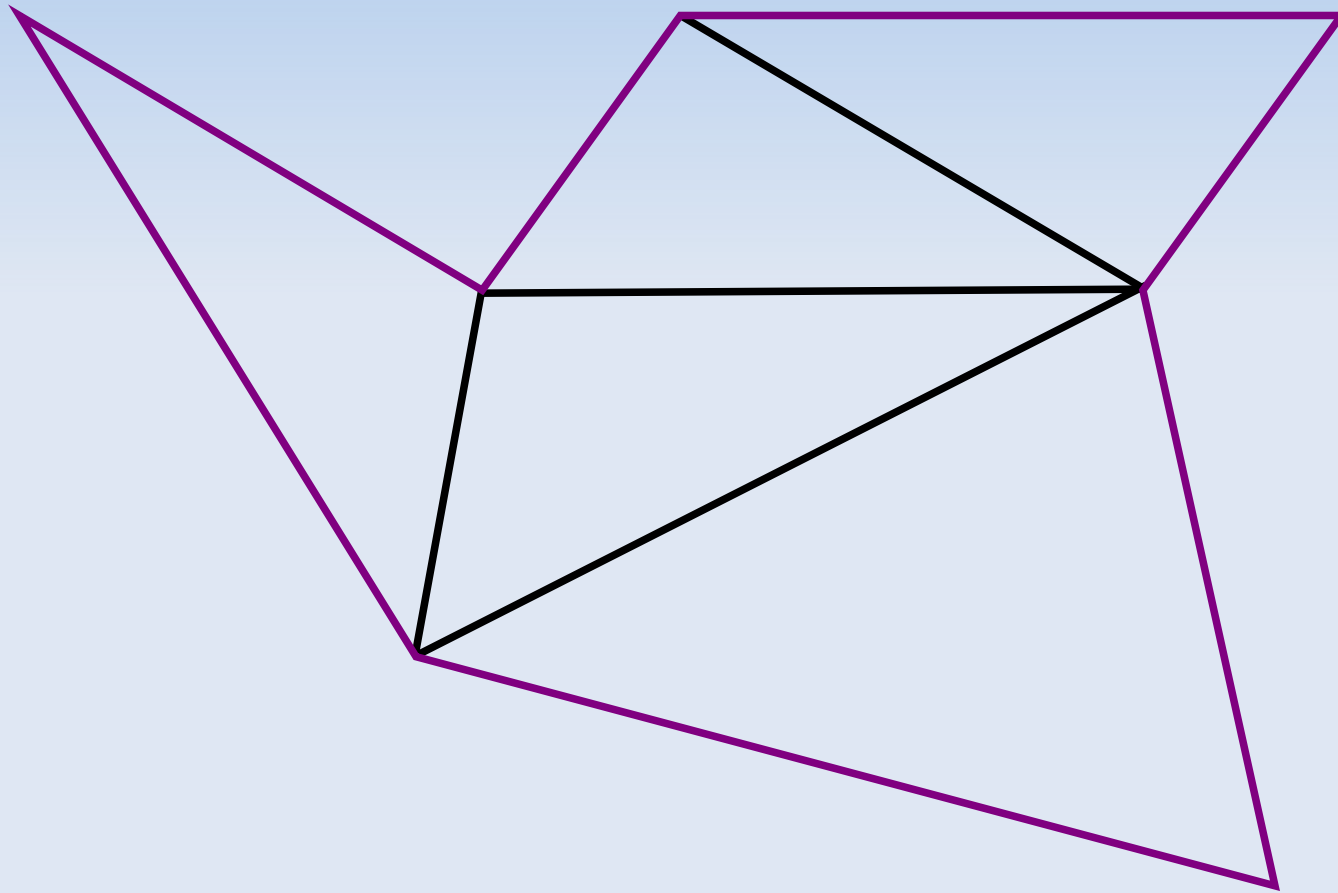
# A simple example: intersection

- Take two regions, and find their intersection



- Approach
  - Triangulation
  - Intersect the triangles

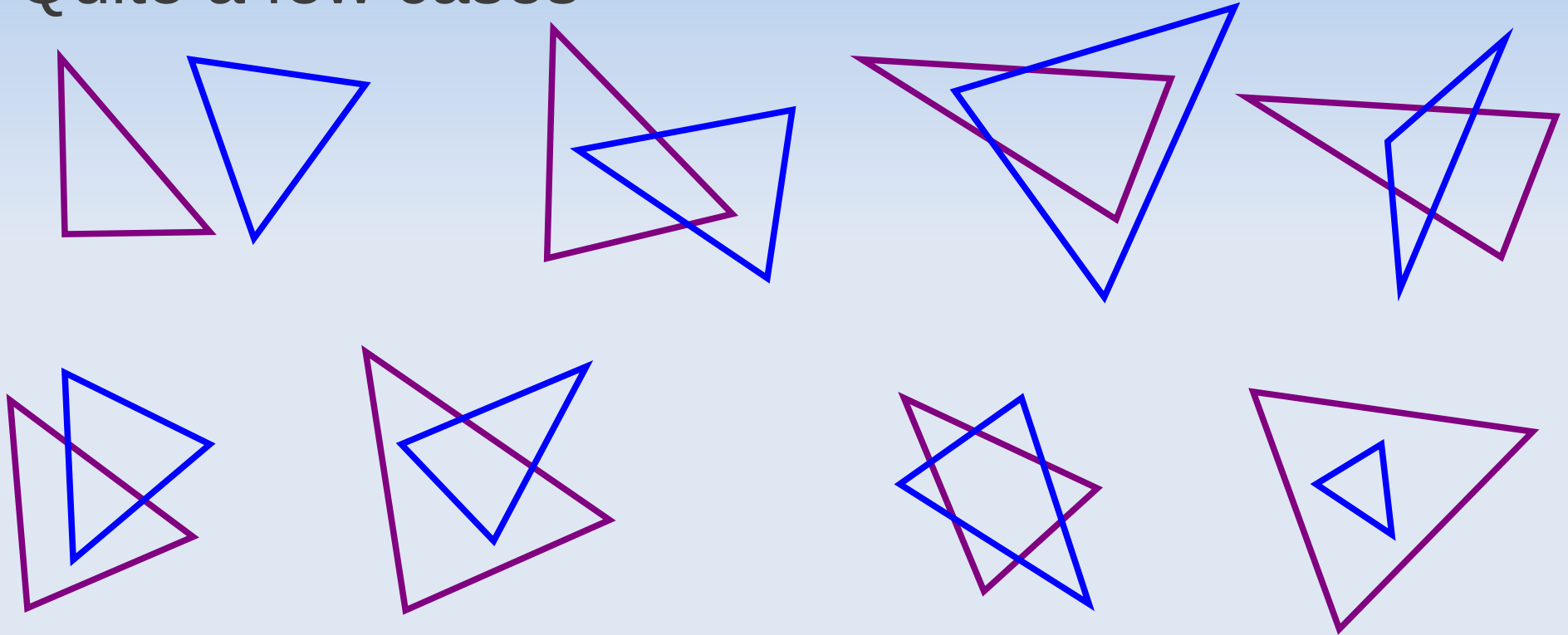
# Triangulation





# Intersection of triangles

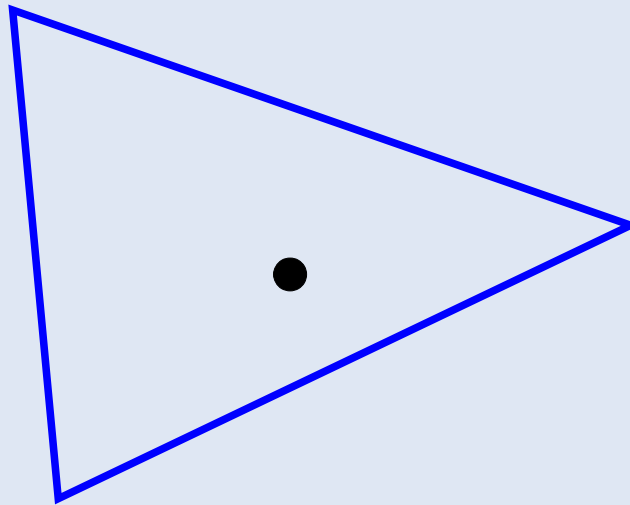
- Quite a few cases



- Plus all the special cases, e.g.,
  - Shared vertex, or vertex on an edge

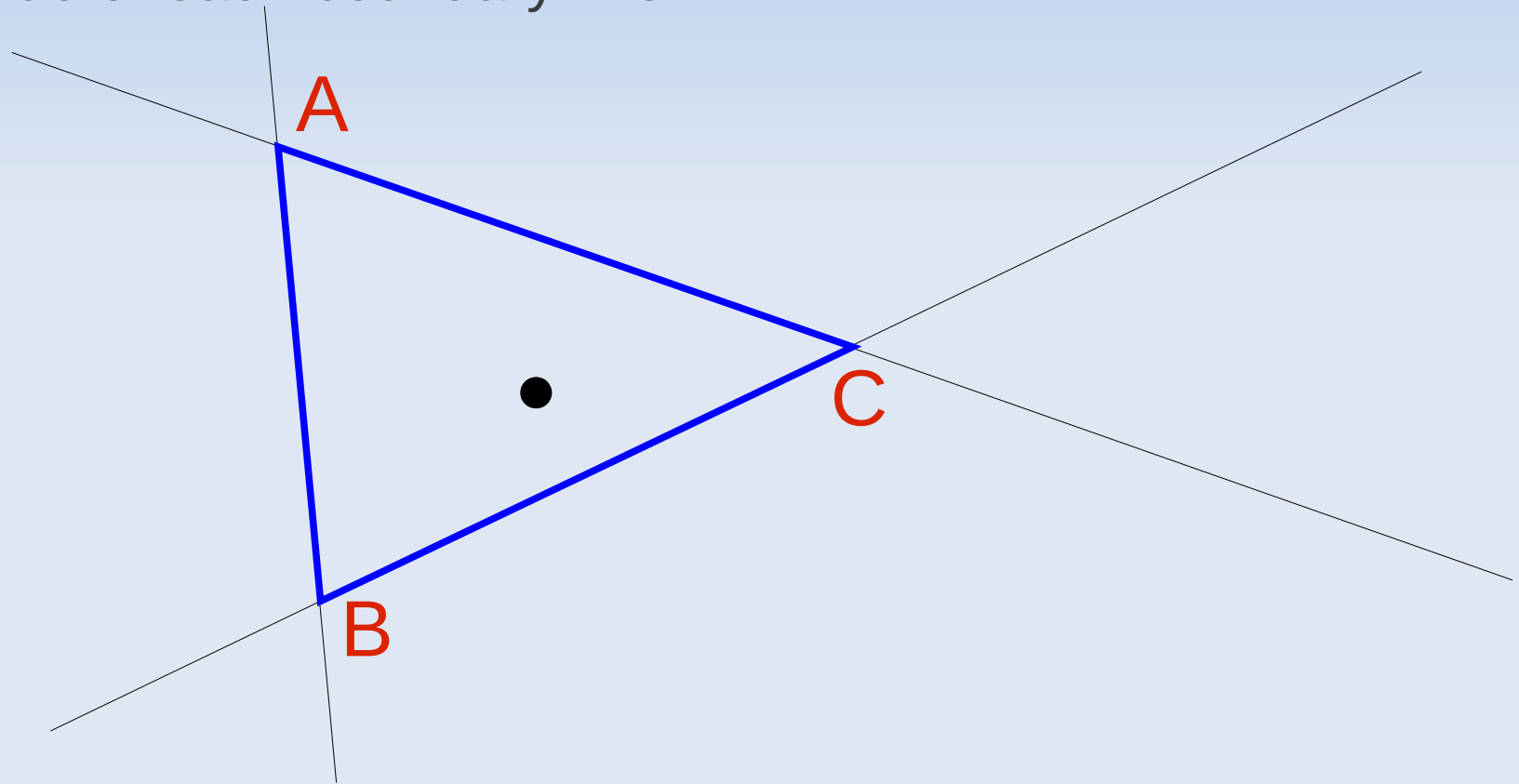
# Triangle intersection

- Important things to check
  - Line intersections
  - Is a point inside, or outside a triangle?



# Point-triangle test

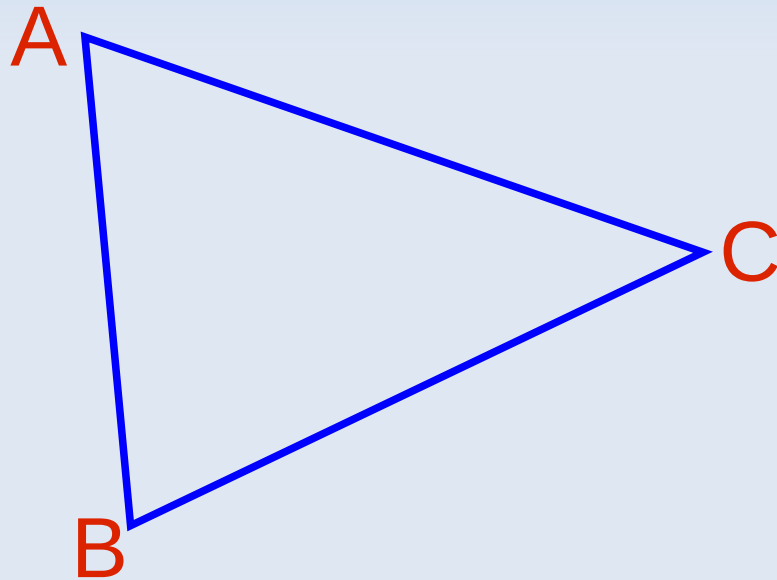
- Test whether a point is inside a triangle by checking its on the correct side of each boundary line



- We can do that with triangle areas

# Area of a triangle

- Many ways to compute the area of a triangle
  - Here is one using the determinant of a matrix

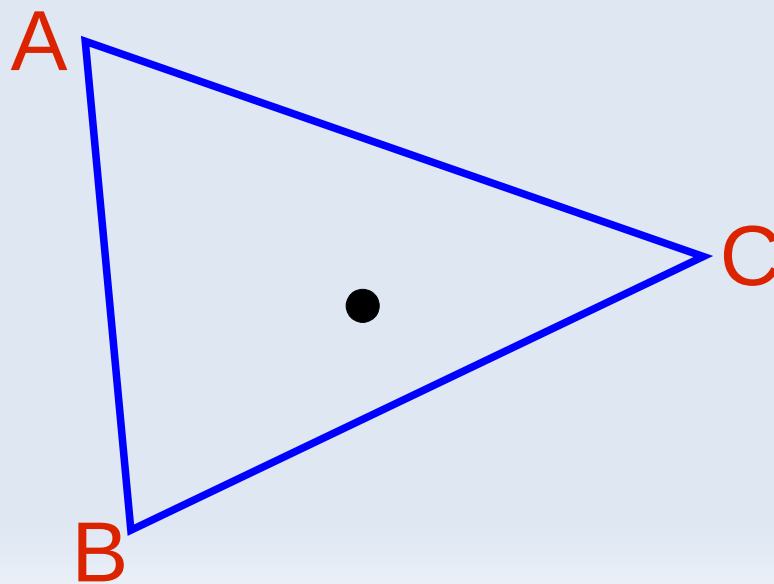


$$\text{Area} = \begin{vmatrix} x_A & x_B & x_C \\ y_A & y_B & y_C \\ 1 & 1 & 1 \end{vmatrix}$$

- Much quicker than using trig
- Order of vertices determines the sign

# Area of triangle

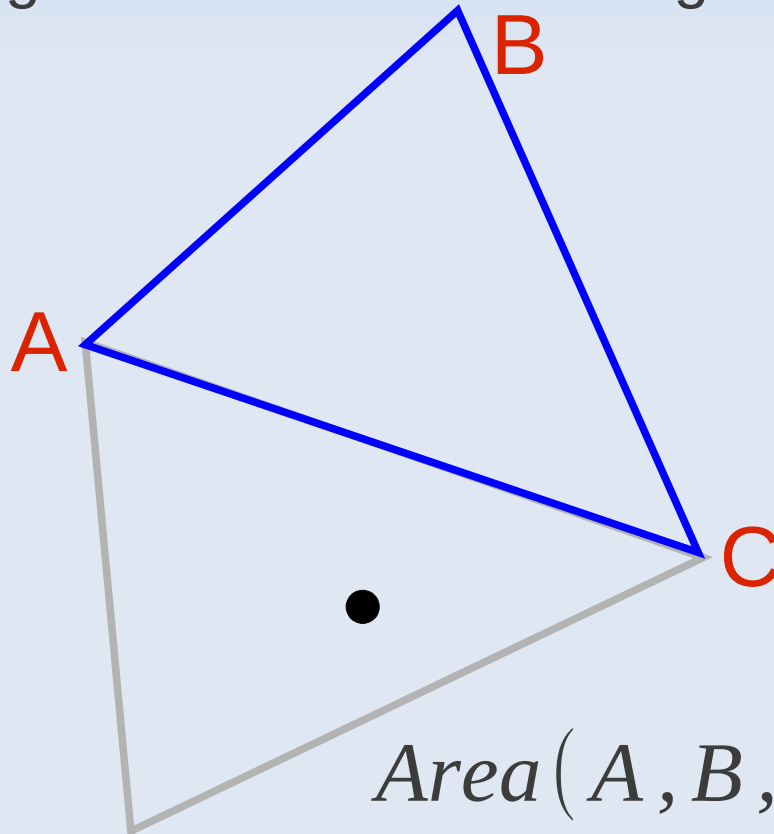
- Area measurement has plus or minus sign depending on the order of the points
  - swapping two columns in a determinant changes its sign



$$Area = \begin{vmatrix} x_A & x_B & x_C \\ y_A & y_B & y_C \\ 1 & 1 & 1 \end{vmatrix}$$

# Area of triangle

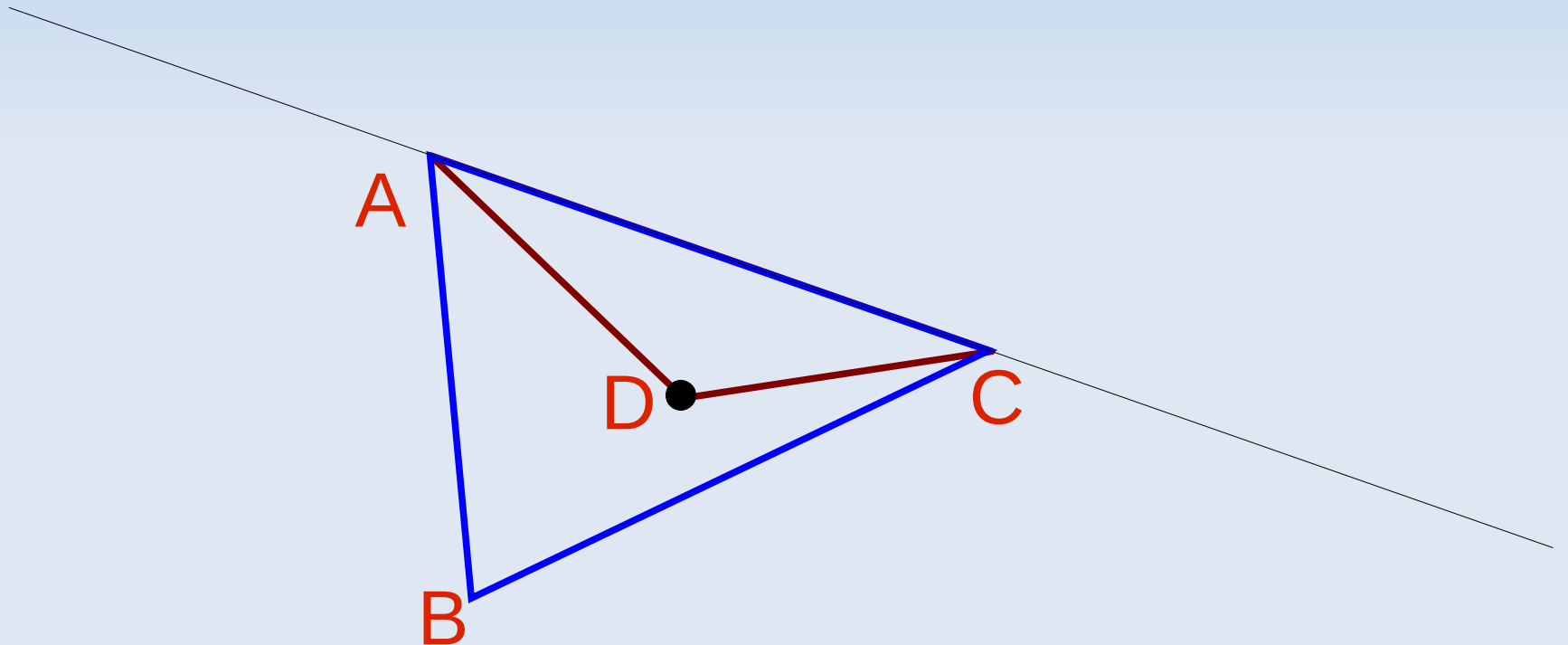
- Area measurement has plus or minus sign depending on the order of the points
  - Changing order of vertices changes the sign of our "Area"



$$\text{Area}(A, B, C) = -\text{Area}(A, C, B)$$

# Point-triangle test

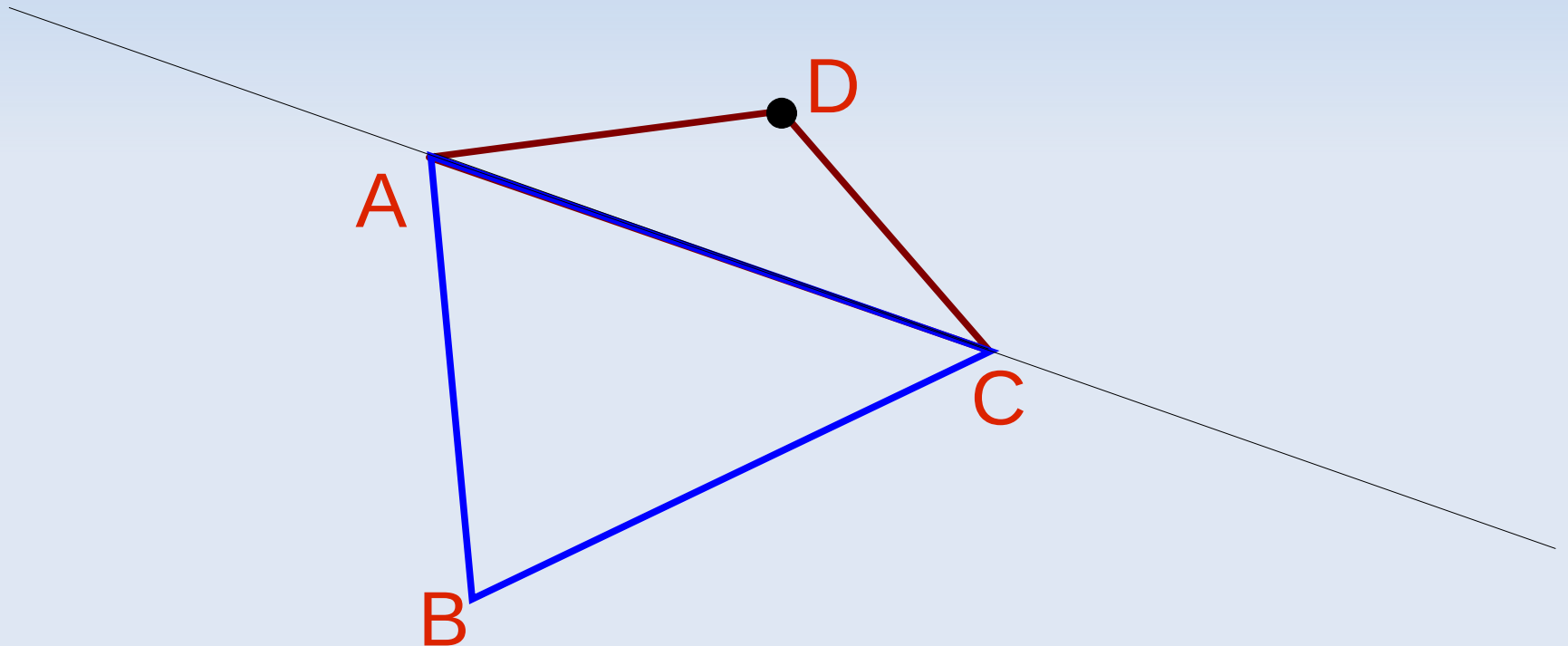
Compare the sign of  $Area(A,B,C)$  and  $Area(A,D,C)$



*We learn if D is on the same side of  $\overline{AC}$  as B*

# Point-triangle test

Compare the sign of  $Area(A,B,C)$  and  $Area(A,D,C)$



*We learn if D is on the same side of  $\overline{AC}$  as B*



# Obviously...

- Obviously this is simplified
  - I need other bits to do intersection
  - I need many other algorithms
- The point is
  - Lots of maths hidden in something like 3D printing
  - Geometry and Linear algebra ++
- Same math used in other areas
  - CGI used in movies
  - Computer game graphics